# The Theory and Empirical Research of Risk Measuring Method Statistics and Portfolio Model

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**Abstract:** The securities market is a high-risk market, which is affected by the national economic situation, policy changes and the market's own laws. Under the background of the rapid development of China's securities investment fund market, the number and scale of funds have been expanding, and market risk has rapidly become the main risk facing China's securities investment fund market. In most frameworks, risk measurement is a function of portfolio returns in all possible ranges. Once the behavior of investors can be expressed numerically, it is actually possible to use different optimization methods to calculate the optimal asset allocation for a particular investor. Investors often make necessary analysis and evaluation on the risk status of the invested objects before and during the investment, and find out management countermeasures that are consistent with their investment objectives and characteristics, so as to reduce possible losses. The globalization and integration of economy and finance, the innovation of financial products and systems, and the popularization of the Internet have increased financial risks day by day.

## 1. Introduction

The securities market is a high-risk market, which is affected by the national economic situation, policy changes and the laws of the market itself. Especially in today's highly irregular securities market in China, there are many unpredicTable factors [1]. In most frameworks, risk measurement is a function of portfolio returns in all possible ranges. With the rapid development of China's stock market, the links between markets or various investment assets are getting closer and closer, and the mutual influence is also getting bigger and bigger [2]. Therefore, the correlation degree and mode between China's financial markets have also changed greatly. Like any kind of investment activity, the risks faced by engineering investment are also huge. In recent years, with the great heat of the real estate market, it has also led to the rise of all kinds of investment in real estate projects. However, it is accompanied by increasingly obvious risks [3]. The violent fluctuations in financial market prices caused by financial reforms and developments in Western countries have made global investors and financial institutions face great risks [4]. Under the background of the rapid development of China's securities investment fund market, the number of funds, especially the number and scale of open-end funds, has continuously expanded, and market risks have quickly become the main risks faced by China's securities investment fund market [5]. Investors invest in the securities market, rely on their own rational judgment, choose investment targets, and expect a certain return on investment, but also face many risks [6].

Whether the model is used to select an investor stock portfolio, a company's asset-liability portfolio, or a bank's hybrid derivatives, the common thread in all models is to take some form of risk metric while maximizing some form of revenue metric. Minimize [7]. The traditional portfolio theory assumes that the combined rate of return is subject to a multivariate normal distribution, which is inconsistent with the characteristics of the return on assets usually expressed as peaks and thick tails [8]. Because the risk of investing in securities may encounter various risks, it will cause losses to investors, thus making the research of securities investment risk of great significance and value [9]. In most frameworks, risk measurement is a function of portfolio returns in all possible ranges. Once the behavior of investors can be expressed numerically, it is actually possible to use different optimization methods to calculate the optimal asset allocation for a particular investor [10]. Prediction and identification of market risks, measurement and monitoring of risks, and dispersion

and avoidance of risks constitute the three pillars of modern risk management theory. The risk of securities investment refers to the possibility that the actual income obtained is lower than the expected income [11]. When the data contains outliers, it will deviate from the normal distribution, so these traditional estimation methods no longer have excellent estimation properties, which requires us to find a statistical method that can have a better evasive effect on outliers [12].

Financial institutions and investors, based on their pursuit of maximum benefits and their special needs for risk management and control, urgently require the application of mathematical and financial theories by means of quantitative management in practice [13]. When individuals or companies focus on lower risks such as extremely low returns or negative cash flows, the exhaustive form of risk measurement cannot accurately reflect their risk preferences [14]. Investors often make necessary analysis and evaluation on the risk status of the invested objects before and during the investment, and find out management countermeasures that are consistent with their investment objectives and characteristics, so as to reduce possible losses [15]. The continuous development and improvement of China's economy has played a very important supporting role in the establishment and development of the securities market, especially for today's stocks, funds, bonds and options [16]. The continuous emergence of these securities investment instruments has also had a profound impact on the current development speed and quality of China's economy, and the corporate financing investment decisions and investors' individual investment ideas will also change accordingly [17]. The changes in economics and financial globalization and integration, the innovation of financial products and systems, and the popularity of the Internet have made financial risks increasingly rampant [18]. The measurement and monitoring of market risk is the foundation and core of the whole risk management theory, and also the frontier and main subject of risk research.

## 2. The Development of Risk Measurement Methods

### 2.1 Average of Opportunity Cost and Transaction Cost

When securities investors invest, they have many different choices on how to allocate the total funds to investment projects with different risks (and without risks). We use combined variance in the mean variance framework and convex utility function in the random programming framework to apply to a set of all possible returns. Academic circles at home and abroad have maintained a high level of attention and conducted in-depth research on the measurement and monitoring of market risks. The great progress of Internet technology, financial informatization, modern financial theory, and the emergence and wide application of financial engineering have enhanced the liquidity of funds in the financial market. Many investors prefer to use quantitative methods for portfolio investment, but common traditional basic analysis and technical analysis are still the mainstream. The securities market is changeable. In order to obtain higher investment returns in these securities markets, investors must make rational decisions according to the specific conditions of the market. Risk and benefit are twin brothers. Investors with different preferences may have different measurement standards and different risk measures [19]. When investors find that they are facing greater risks in the process of the continuous development of financial markets, they will try to find ways and means to avoid and disperse risks. Risk is defined as the change of market value or return of securities or portfolios. The risk attributes of securities or portfolios can be expressed by the density function of probability distribution.

When using combinatorial model algorithm to analyze problems, it is usually assumed that the real-time utility function of the cardinal number does not change with time, that is to say, the welfare of the same income is the same in different periods. In reality, the preferences of decision makers will change irregularly over time. When the fund manager changes, the new fund manager's judgment on macroeconomic changes and changes in the competitive environment within the industry is certainly different from that before the change. If the original time series has long-term memory, the order of data is very important. If the investor's utility function is not logarithmic, the calculation of relevant results is a problem to be solved. According to the analysis, it is almost

impossible to get an analytical solution. According to the price time series of the past period of market factors, the actual changes of the price level of the past period of market factors are calculated. The above calculation is realized by language programming. The calculation results are shown in Table 1.

Table 1 Analysis and related parameter estimation results

|  | Constant term | Coefficient term | Fractional dimension | Correlation scale |
|---|---|---|---|---|
| Daily rate of return | 0.375 | 0.782 | 1.546 | 0.342 |
| Weekly rate of return | 0.342 | 0.734 | 1.233 | 0.317 |
| Monthly rate of return | 0.327 | 0.765 | 1.317 | 0.286 |

Securities investment is a dynamic investment selection process. In an environment where the market is not yet fully developed, and at the same time there are many factors outside the securities market itself, which makes it difficult for investors to predict the future trend of the securities market. When the rate of return does not obey the normal distribution, the mean and variance cannot describe all the characteristics of the distribution. Portfolio method is often used in financial industry and efficient researchers, especially in some financial industries such as securities market [20]. For the mean-variance model previously discussed, it does not measure risk from the perspective of an investor, and the investment risk it obtains is unique. The mean-entropy optimization model of the bond portfolio provides an auxiliary tool for investors to make decisions in the underdeveloped environment of the securities market, and also explores a new path for research on investment portfolios in this environment. The joint distribution must consider not only the marginal distribution of each risky asset, but also the relevant structure between the distribution of securities yields [21]. Opportunity costs are a potential and not actually incurred cost, which is different from the cost of expenses or expenses that we usually recognize. Investors can generally be divided into risk aversion, risk appetite and risk neutrality. For different investors, the construction of the portfolio and the ability to withstand risks are different.

## 2.2 Average Value of Portfolio Optimization

Portfolio mainly refers to various assets owned by individual or institutional investors. Securities investment portfolio is an investment method in which funds are invested in several kinds of securities according to a certain proportion. As the core problem of kernel estimation, bandwidth and kernel function selection need to be analyzed emphatically. In addition, bandwidth is also the influencing factor for variance and deviation of kernel estimation. Securities investment is a complicated and risky financial activity, which may not only bring rich returns to investors, but also make investors suffer huge losses. The semi-variance expression is relatively complex. According to it, the semi-covariance matrix needs to be calculated when making portfolio selection. The mean semi-variance model needs twice as much data as the mean one-way difference model, which brings great difficulties to the actual calculation. Traditional portfolio theory is the core and one of the classic theories in modern western investment theory, and its function is significant and very important. Since financial risk managers are most concerned about the risks occurring at the left end of the yield distribution, the risk measurement method that is used in risk management is the tail risk measurement based on the quantile [22]. The global integration of the financial market, especially the international securities market, has increased the risk and complexity of the current securities market. More and more investors have fully realized the importance of establishing portfolio and using diversified investment to diversify risks.

In order to get a definite result, it is further assumed that the investor has a logarithmic utility function and the return on riskless assets is constant. All intertemporal selection theories assume diminishing marginal utility and positive time preference rates. These two hypotheses play an

important role in cross-period selection, and the diminishing marginal utility promotes the time distribution of investment income to be dispersed over time. If the time series obeys the normal distribution, the normal probability distribution map will show a straight line tilting to the upper right through the origin. On the contrary, a large number of observations are far from the normal distribution probability line. The greater the volatility of risk-free asset prices, the smaller the information value. The inconsistency of expected return leads to the inconsistency of time discount rate of financial institutions. With the time discount rate decreasing, hyperbola is steeper than hyperbola, which means that the degree of inconsistency of time preference is more obvious. As shown in Figure 1.
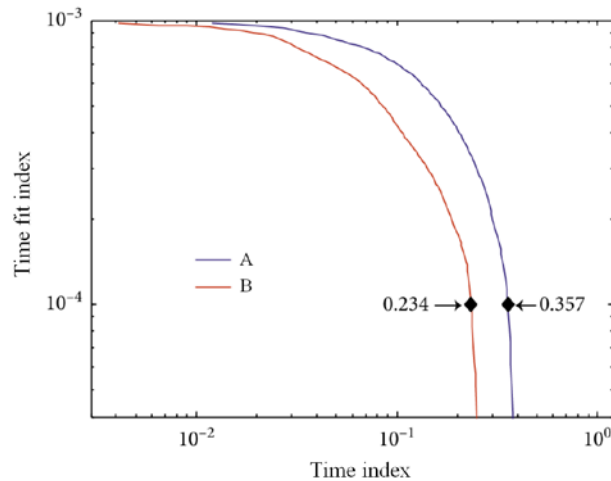


Fig. 1 Time inconsistency curve

The parameter method assumes that the change of risk factor's return follows a certain distribution. Then the historical data of the change of risk factor's return are analyzed by statistics, and the parameter value of the return distribution is deduced. In today's capital market, short-selling mechanism, opportunity cost, transaction cost, risk preference and other factors of securities investment always affect investors'trading decisions. We aim at the relatively perfect development of the securities market, which not only includes the integrity and credibility of the past statistical data, but also assumes that there will be no unexpected events in the future securities market. In the securities market full of risks and opportunities, both individual and institutional investors always take the safety and liquidity of investment funds as the premise when they invest in securities. Reasonable use of investment funds to achieve the purpose of less risk and higher returns. The estimated value of the maximum potential loss faced by a certain financial asset or portfolio within a certain holding period and a certain statistical confidence level. The normalized residual product delayed by one period has a significant effect on the correlation coefficient, while the normalized residual product delayed by one period has little effect on the correlation coefficient for the financial variables of other portfolio assets. In the process of securities trading, the transaction cost is generally a two-way charge, that is, a kind of cost that needs to be borne when the transaction is bought and sold.

## 3. Application of STable Distribution in Investment Portfolio

### 3.1 Application in Selecting Stock Market Investment

Risk aversion investors really want to avoid very low returns. They are welcome to the parts that exceed the expected returns. Therefore, variance can not fully weigh the relationship between the expected returns and risks of the portfolio. Normal solution method is only suiTable for solving single securities. In the case of several securities investment, we usually need to set the weight of each securities first, and then calculate the portfolio income data through the historical data of each securities, so that the distribution parameters of portfolio income can be estimated and VaR can be obtained. VaR risk measures include all information about risk factors under normal market

conditions and the maximum possible losses resulting therefrom. Traditional portfolio investment is quite different from individual asset investment. The main purpose of the former is to diversify risk. It is assumed that all engineering investors satisfy the condition of pursuing profit maximization under certain investment risk. In other words, it means that investors satisfy the hypothesis of rational economic man. Because the portfolio model is to solve the portfolio with the lowest risk under the condition of a certain return, that is, to choose the portfolio with the lowest risk among the portfolios with the same return rate, the risk of the portfolio depends on the variance of the portfolio. Financial institutions often use drag-and-drop method to measure VaR. When the relationship between asset value and market risk factors is non-linear and non-fixed, it is very effective to measure VaR by simulation.

In general, the nonlinear filtering technique is used to obtain the optimal estimate of the average return rate of risky assets. Then, instead of the usual more complicated stochastic dynamic programming method, the optimal investment strategy was quickly obtained. The unit test results show that each variable is a first-order single integer sequence, which meets the premise of cointegration test. Check whether there is a long-term equilibrium relationship between relevant variables. According to the model lag time selection criteria. The inspection results are shown in Table 2.

Table 2 Cointegration test results

| Eigenvalues | Trace estimate | Threshold |
|---|---|---|
| 1.531 | 55.77 | 25.69 |
| 0.843 | 43.83 | 36.49 |
| 1.176 | 44.55 | 29.42 |
| 0.867 | 26.45 | 48.28 |

For the time trajectory of the financial time series, the time series with long-term correlation is:

$$U_{ij} = \frac{H_{ij}}{\sqrt{\sum_{t=1}^{k} H_{it}^2}}, i = 1,...,n, j = 1,...,k \tag{1}$$

Time series have a long-term "memory" effect. The previous observations have long-term effects on the observations of the latter phase, and this effect is measured by the associated scaling function:

$$I(X;Y) = \sum_{y \in Y} \sum_{x \in X} p(x,y) \log(\frac{p(x,y)}{p_1(x)p_2(y)}) \tag{2}$$

Sequences are biased random walks with persistence and long-term memory effects. The statistic for this hypothesis test is n:

$$n = \sum_{i-1}^{R} P_l W_{l,j} + b \tag{3}$$

To describe the impact on the future, an indicator of relevance is introduced:

$$E_p = \frac{\sum(t_{pi} - o_{pi})^2}{2} \tag{4}$$

Variance represents a fluctuation in which the actual return deviates from the average return. The greater the fluctuation, the greater the uncertainty of the actual return, regardless of whether the actual return is higher or lower than the average return. Before the theory of portfolio was formally put forward, some scholars have mentioned the concept of portfolio in their papers, but due to the lack of comprehensive theoretical support, there is also a lack of systematic support. The return rate of risky securities has obvious non-normal distribution characteristics such as skewness and excessive peak. It has become a consensus in academic circles that the return rate of risky securities

does not obey normal distribution. In the selection of several investment schemes, assuming that investors need to determine an optimal investment scheme, they need to abandon other investment schemes or the profits that investment opportunities may bring. Assuming that the yield sequence obeys a specific distribution, and the yield sequence satisfies independent and identical distribution at the same time, it is relatively simple and convenient, so it has been widely used in practice. Due to various factors that bring risks, investors are confronted with many unpredicTable risks when investing, which shows that variance alone cannot quantitatively represent all risks. To determine the investment objectives, investors need to consider their risks when constructing their investment portfolios if they want to obtain more sTable returns.

## 3.2 Portfolio Model Based on Stable Distribution

Under the condition that the return rate of risky securities obeys non-normal distribution with skewness and excessive peak, variance is not suiTable to measure the risk of portfolio. In order to simplify the model, it is necessary to ensure that capital is under certain constraints while eliminating the cost of raising capital when determining the optimal investment portfolio of investment enterprises. The distribution of time series of return on financial assets does not satisfy the normal hypothesis and has a significant peak and heavy tail characteristic. On the other hand, its fluctuation has obvious characteristics of agglomeration and time-varying. If an effective portfolio has higher expected return than other portfolios, it also has higher risk characteristics than other portfolios. Because investors may change their investment objectives over time or other factors, they should revise the previous portfolio and evaluate the investment effect reasonably in order to achieve the optimal state. When variance is used to measure the risk of a portfolio, the correlation structure between the distribution of securities returns is measured by covariance or Pearson linear correlation coefficient. Considering that when the return rate of risk-free securities in non-portfolio is lower than that of risk-free securities investment, investors will obviously choose risk-free securities to invest. Therefore, the opportunity cost faced by investors at this time should be risk-free securities.

The chaos analysis method of exchange rate determination is still in the early stage of development, but this method has important theoretical and practical significance. Traditional exchange rate determination theory focuses on the sTable state of exchange rate, while structural equation model makes people realize that the periodic and chaotic movement of exchange rate is also possible. At present, the RMB exchange rate is not completely free to float and is often interfered by the central bank. It is not a freely tradable variety. For most financial event sequences, the variance does not follow the assumption that most existing time series models are constant, but has time-varying characteristics. Mean structure mutation point refers to the point where the time series is divided and the mean value of the time series changes suddenly before and after the time series is divided. In recent years, the methods of exchange rate time series analysis have been continuously innovated and developed. At present, the most widely used nonparametric method belongs to the category of data mining research. The exchange of foreign exchange by ordinary residents in the country is still subject to relatively large control, and they cannot generally participate in foreign exchange transactions. The existence of agency problem makes the time preference of asset managers inevitably internalized in the investment decision-making activities of financial institutions. The comparison data of financial situation risks are shown in Table 3. The data relationship between risk weight value and evaluation value of financial situation is shown in Figure 2.

Table 3 Financial situation risk comparison data results

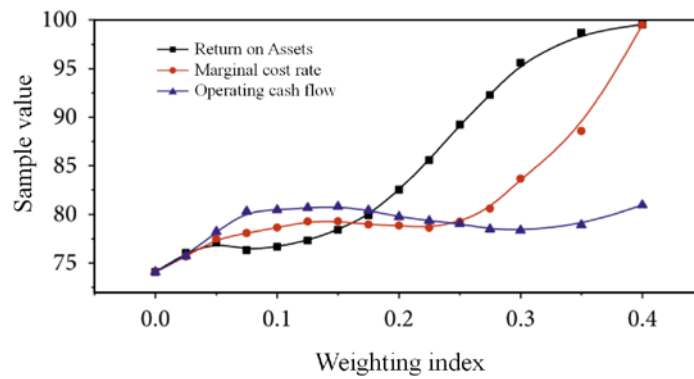|  | Return on assets | Marginal cost rate | Operating cash flow |
|---|---|---|---|
| Return on assets | 1 | 0.87 | 0.64 |
| Marginal cost rate | 0.76 | 1 | 0.59 |
| operating cash flow | 0.55 | 0.62 | 1 |

Fig. 2 Relationship between financial position risk weight value and evaluation value data

Investors are risk-averse, that is to say, the goal pursued by investors is to maximize returns under a given risk condition or minimize risks under a given return level. To construct an investment portfolio, after determining the investment objectives, investors should choose a portfolio that is consistent or consistent with the investment objectives. Under the condition of non-normal distribution, the linear correlation coefficient cannot correctly measure the correlation between random variables. When the return rate of the risk securities in the non-portfolio is greater than or equal to the risk-free securities investment, it is obvious that the single risk securities with the highest return rate in the non-portfolio will become the opportunity cost of investors. Investors seek to maximize the expected utility of each period of wealth. Investors have a one-cycle perspective, that is, investors have a definite investment interval, and only consider the benefits obtained during this period. The return data of securities obey normal distribution. Risk can be expressed by variance of return rate. Return can be expressed by expected return rate. Investors only pay attention to expected return rate and variance of investment when making decisions. For risk-averse rational investors, even if they abandon the original portfolio plan to choose other non-sister-in-company risk securities for investment, they will not only invest in the most profiTable risk securities.

## 4. Conclusion

Firstly, this paper analyzes and summarizes the measurement of existing securities investment risks, points out the shortcomings of these methods, and puts forward a new measurement method of securities investment risks on the basis of in-depth discussion on the essential attributes of securities investment risks. The purpose for investors to make securities investment decisions is to obtain more profits with as few risks as possible. However, in the actual investment transaction process, it is ineviTable to involve fees or costs such as taxes, transaction commissions, etc. The theoretical assumption of measuring investment risks by using lower partial moments is more relaxed, and it can effectively reflect the psychological characteristics of investors for investment risks. The empirical results show that the variance, variance contribution rate and cumulative variance contribution rate obtained by the robust method can better reflect the actual situation than those obtained by the traditional method before and after rotation, and the robust method can better reflect the outlier value. Resistance is stronger. It is necessary to analyze the correlation among multiple variables in the financial market and find the appropriate variables, measurement methods or statistics needed in the modeling. With the deepening of people's understanding of the nature of investment risk, new risk measurement methods and portfolio optimization models will continue to emerge.

## References

[1] Belles-Sampera J, Guillén, Montserrat, Santolino M. GlueVaRrisk measures in capital allocation applications [J]. Insurance: Mathematics and Economics, 2014, 58:132-137.

[2] Cheung K C, Sung K C J, Yam S C P, et al. Optimal reinsurance under general law-invariant

risk measures [J]. Scandinavian Actuarial Journal, 2014, 2014(1):72-91.

[3] Tsukahara, H. Estimation of Distortion Risk Measures[J]. Journal of Financial Econometrics, 2014, 12(1):213-235.

[4] Natasha S, Summers L H. Understanding Bank Risk through Market Measures[J]. Brookings Papers on Economic Activity, 2016, 2016(2):57-127.

[5] Chuang Y, Schechter L. Stability of experimental and survey measures of risk, time, and social preferences: A review and some new results[J]. Journal of Development Economics, 2015, 117:151-170.

[6] Ewald C O, Yor M. On increasing risk, inequality and poverty measures: Peacocks, lyrebirds and exotic options[J]. Journal of Economic Dynamics and Control, 2015, 59(2):22-36.

[7] Pradkhan E. Impact of culture and patriotism on home bias in bond portfolios[J]. Review of Managerial Science, 2016, 10(2):265-301.

[8] Bams D, Blanchard G, Lehnert T. Volatility measures and Value-at-Risk[J]. International Journal of Forecasting, 2017, 33(4):848-863.

[9] Gerlach R, Walpole D, Wang C. Semi-parametric Bayesian tail risk forecasting incorporating realized measures of volatility[J]. Quantitative Finance, 2016:1-17.

[10] Nainggolan Y, How J, Verhoeven P. Ethical Screening and Financial Performance: The Case of Islamic Equity Funds[J]. Journal of Business Ethics, 2016, 137(1):83-99.

[11] Kim S H, Lee K H. Pricing of liquidity risks: Evidence from multiple liquidity measures[J]. Journal of Empirical Finance, 2014, 25(Complete):112-133.

[12] Dermine, Jean. Basel III leverage ratio requirement and the probability of bank runs[J]. Journal of Banking & Finance, 2015, 53:266-277.

[13] Buch C M, Eickmeier S, Prieto E. In search for yield? Survey-based evidence on bank risk taking[J]. Journal of Economic Dynamics and Control, 2014, 43:12-30.

[14] Gonzalez M, Rodriguez A, Stein R. Portfolio performance under reference-day risk[J]. Investment Analysts Journal, 2016:1-12.

[15] Assa H. On Optimal Reinsurance Policy with Distortion Risk Measures and Premiums[J]. Papers, 2014, 61(61):70-75.

[16] Brandtner M, Kürsten, Wolfgang. Solvency II, regulatory capital, and optimal reinsurance: How good are Conditional Value-at-Risk and spectral risk measures?[J]. Insurance: Mathematics and Economics, 2014, 59:156-167.

[17] Zheng Y, Cui W. Optimal reinsurance with premium constraint under distortion risk measures[J]. Insurance: Mathematics and Economics, 2014, 59:109-120.

[18] Baek S, Cursio J D, Cha S Y. Nonparametric Factor Analytic Risk Measurement in Common Stocks in Financial Firms: Evidence from Korean Firms[J]. Asia-Pacific Journal of Financial Studies, 2015, 44(4):497-536.

[19] Valsamakis A C. The use of comprehensive cash flow analysis as a basis for risk measurement and determination of debt capacity[J]. Investment Analysts Journal, 2015(18):59-60.

[20] Xidonas P, Mavrotas G. Comparative issues between linear and non-linear risk measures for non-convex portfolio optimization: evidence from the S&P 500[J]. Quantitative Finance, 2014, 14(7):1229-1242.

[21] Chen Z, Liu J, Hui Y. Recursive risk measures under regime switching applied to portfolio selection[J]. Quantitative Finance, 2017:1-20.

[22] Brandtner, Mario. Expected Shortfall, spectral risk measures, and the aggravating effect of background risk, or: risk vulnerability and the problem of subadditivity[J]. Journal of Banking & Finance, 2018, 89:138-149.